

# Quantum computing in control and optimization

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April 13, 2007

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# Introduction

- In quantum computing, information is manipulated not discretely, in the classical way, as a series of zeros and ones (bits), but as continuous superposition (qubits) where the number of possible states is vastly greater
- Quantum Computers will be able to make many computations simultaneously
- Quantum processors are not yet developed, but this is just a matter of time
- At the moment there is great interest for controlling quantum processes

# Introduction

Problems of controlling microprocesses and quantum ensembles were posed and solved for:

- plasma and laser devices
- particle accelerators
- nuclear power plants
- automation units

The problem of controlling quantum states was in fact brought forth due to the rise of quantum mechanics.

# Evolution of Quantum Systems

- The goal of our work is using the controlled transition of Quantum Dynamical System (QDS) from the given initial state to the desirable terminal state in predefined time
- Generally speaking, controls that achieve their objective in minimum time are desired to minimize dissipative effects associated with residual couplings to the systems environment
- From a mathematical perspective, many of these problems reduce to time-optimal control of bilinear systems evolving on finite or infinite dimensional Lie groups
- Although bilinear control problems have previously been studied in great detail, rich new mathematical structures can be found in quantum problems.

# Evolution of Quantum Systems

- The problem of controlling a Quantum Dynamical System (QDS) can be stated as follows:

On an interval  $0 \leq t \leq T$  we want to find a certain control function  $u(t) = \{u_j(t)\}$ ,  $j = 1, 2, \dots, n$  from the set of the admissible controls  $U$ , which makes it possible to compose the evolution operator  $S_u(T, 0)$  realizing the mapping

$$\alpha_u(0) \xrightarrow{S_u(T,0)} \alpha_u(T) \quad (1)$$

under the constraint

$$i \frac{d}{dt} S = H(u, t) S, \quad S(0) = I, \quad h = 1. \quad (2)$$

- In quantum mechanics The generator of evolution  $H(t)$  is represented by the Hamiltonian
- Now let us assume that  $H$  generates a finite dimension dynamic Lie algebra

$$H(t) = \sum_{i=1}^n u_i(t)L_i. \quad (3)$$

- The totality of operators  $\{L_i\}$  generates a basis of an  $n$ -dimensional Lie algebra, and the  $u_i(t)$  are scalar functions of time with complex values. It follows from the classical Frobenius theorem that the solution (2) is at least local and can be represented as

$$S = \prod_{i=1}^n \exp(g_i(u, t)L_i). \quad (4)$$

- By using the the *Levy–Maltsev theorem* we can decompose  $S$  as follows:

$$S = S_{\mathfrak{R}} S_R, \quad (5)$$

where

$$i \frac{d}{dt} S_{\mathfrak{R}} = H_{\mathfrak{R}}(t) S_{\mathfrak{R}}, \quad S_{\mathfrak{R}}(0) = I, \quad (6)$$

$$i \frac{d}{dt} S_R = S_{\mathfrak{R}}^+ H_R(t) S_{\mathfrak{R}} S_R, \quad S_R(0) = I, \quad (7)$$

$$H(t) = H_{\mathfrak{R}}(t) \oplus H_R(t), \quad (8)$$

and the operators  $H_{\mathfrak{R}}$  and  $H_R$  generate Lie algebras  $\mathfrak{R}$  and  $R$  correspondingly.

- We can then further decompose  $S_{\mathfrak{X}}$  as follows:

$$S_{\mathfrak{X}} = \prod_{i=1}^n S_i, \quad (9)$$

- where each co-factor satisfies the equation of type (6) with the Hamiltonian  $H_i$ , which generates the corresponding ideal  $\mathfrak{X}_i$ . Moreover,

$$H_{\mathfrak{X}}(t) = H_1(t) + \cdots + H_k(t). \quad (10)$$

## Example: The $SU(2)$ group

- The group  $SU(2)$  is the fundamental group in the interaction theory between radiation and a substance
- The Hamiltonian in this case is:

$$H(t) = u_0(t)L_0 + u^*(t)L_- + u(t)L_+, \quad (11)$$

- where  $u$  and  $u_0$  are differentiable complex-valued functions of time and the generators  $SU(2)$  are written in the spherical basis
- Then the solution is given by:

$$S = \exp \left( g_0 - i \int_0^t u_0(\tau) d\tau \right) L_0 \exp g_- L_- \exp g_+ L_+, \quad (12)$$

- Now let's consider a non-relaxing  $N$ -level atom with non-singular non-equidistant spectra as the QDS

$$H_0|N, n\rangle = \omega_n|N, n\rangle, \quad n = 1, \dots, N. \quad (13)$$

- This atom interacts coherently with the modulated polychromatic laser field

$$E(t) = \sum_{k=1}^{N-1} \sum_{\ell=2}^N \operatorname{Re} E_{k\ell}(t) \exp j(\omega_{k\ell}t + \varphi_{k\ell}), \quad k < \ell, \quad (14)$$

- which consists of  $M (= 1, 2, \dots, N(N - 1)/2)$  components. We assume that the component  $(k, \ell)$  with amplitude  $E_{k\ell}$ , central frequency  $\omega_{k,\ell}$  and phase  $\varphi_{k,\ell}$  excites only one transition between the stationary states  $|N, k\rangle$  and  $|N, \ell\rangle$ .

- The Hamiltonian of the interaction of an atom with the field in the dipole approximation has the form

$$H_{\text{int}}(t) = -E(t)d, \quad (15)$$

- where  $d$  is the operator of the electric dipole moment of an atom. The operator  $H_{\text{int}}$  depends explicitly on time and control and it is finite on the time interval  $[0, T]$ .
- Then, the non-stationary Schrödinger equation

$$i \frac{d}{dt} |\Psi(t)\rangle = (H_0 + H_{\text{int}}(t)) |\Psi(t)\rangle \quad (16)$$

- generates the following system of  $N$  equations :

$$i\dot{c}_k = \sum_{\ell \neq k=1}^N u_{k\ell}(t) \exp[i\Delta_{k\ell}(t) + i\varphi_{k\ell}] c_\ell \quad (17)$$

- The vector-function of occupancy is the controllable value

$$P = \{P_1(t), \dots, P_N(t)\}, \quad P_i(t) = |C_i(t)|^2, \quad i = 1, \dots, N, \quad (18)$$

- which is transformed according to the evolution matrix

$$P(T) \equiv |S(u, T)|^2 P(0). \quad (19)$$

# Controlling quantum-mechanical cellular dynamotons

## Definition

By a neural dynamical system, (neural dynamaton or ND), we mean a complex dynamical system in which the nodes are all identical copies of a single controlled Hamiltonian dynamical scheme, the standard cell (Hiebeler and Tater, 1997).

## Quantum control lattices

We consider the cellular dynamaton (CD) in which every cell is considered as the controllable finite-dimensional Hamiltonian system having the form

$$\begin{aligned} \partial_t x^k &= \mathcal{F}_H^k(x^k, a^k, u^k), \quad x^k \in M^k, x^k(0) = x_0^k, \\ \partial_t y_j^k &= -H_j^k(x^k, a^k), \quad j = 1, \dots, m, \quad k = 1, \dots, N, \\ u^k &= (u_1^k, \dots, u_m^k) \in \Omega \subset \mathbb{R}^m, \end{aligned} \quad (20)$$

If there exists the Hamiltonian

$$H^k(x^k, a^k, u^k) = H_0^k(x^k, a^k) - \sum_{j=1}^m u_j^k H_j^k(x^k, a^k),$$

then we have the system of equations

$$\begin{aligned}
 \dot{x}^k &= g_{H_0}^k(x^k, a^k) + \sum_{j=1}^m u_j^k g_{H_j}^k(x^k, a^k), \quad x^k(0) = x_0^k, \quad x^k \in (M^{2n}, \omega), \\
 y_j^k &= -\frac{\partial H^k}{\partial u_j^k}(x^k, u^k), \quad j = 1, \dots, m, \quad k = 1, \dots, N, \\
 u^k &= (u_1^k, \dots, u_m^k) \in \Omega \subset \mathbb{R}^n.
 \end{aligned} \tag{21}$$

# Quantum optimization

- For this quantum computing is very important for the future of Global Optimization
- There is potential to increase the efficiency of stochastic optimization methods.
- In [4] we describe an quantum algorithm for Global minimization

# Conclusion

- This work deals with the progress made on the optimal control of quantum systems.
- We pointed out the significance of lattice quantum search algorithm for global optimization
- Lattice search can provide the basis for implementing adaptive global optimization problems
- One such quantum algorithm introduced by Yatsenko [4] for finding global minima.